Macroeconomics II

10. Technological Progress and Growth

BSc in Economics

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The production function with technological progress

- Aggregate production function F:

$$Y = F(K, N, A) = F(K, AN)$$

where Y is output, K is capital, N is labor, and A is the state of technology.

- The relationship between output *per* effective worker and capital *per* effective worker can be derived as:

$$\frac{Y}{AN} = F\left(\frac{K}{AN}, 1\right)$$

- Or if we define $f(K/AN) = F(K/AN, 1)$, we have:

$$\frac{Y}{AN} = f\left(\frac{K}{AN}\right)$$

Saving and Investment

- Given that public saving $(T - G) = 0$, we observe that $I = S$.
- Assuming a saving function of the form $S = sY$, where s represents the saving rate (a parameter), it follows that $I = sY$.
- Dividing both sides by the number of effective workers, AN, yields:

$$\frac{I}{AN} = s\frac{Y}{AN} = sf\left(\frac{K}{AN}\right)$$

Interactions between output and capital

- Previously, we established that capital remains constant when investment matches the depreciation of existing capital stock.
- Now, in order to sustain a specific level of capital *per* effective worker, the change in K must be proportional to the increase in the number of effective workers, AN, and to capital depreciation.
- The required level of investment to uphold the capital *per* effective worker is expressed as:

$$I = \delta K + (g_A + g_N)K \Leftrightarrow I = (\delta + g_A + g_N)K$$

- The investment amount required *per* effective worker to maintain capital *per* effective worker is:

$$\frac{I}{AN} = (\delta + g_A + g_N)\frac{K}{AN}$$

Dynamics of Capital and Output

- In the steady state of the economy, capital and output *per* effective worker remain constant:

$$sf\left(\frac{K}{AN}\right) = (\delta + g_A + g_N)\frac{K}{AN}$$

- Because effective labor grows at a rate of $(g_A + g_N)$, output growth in the steady state must also equal $(g_A + g_N)$; the same principle applies to capital.
- The steady state is also called a state of balanced growth: output, capital and effective labour grow ”in balance” at the same rate.
- The growth rate of Y/N equals the growth rate of Y minus the growth rate of N. Thus, the growth rate of Y/N is given by $(g_Y - g_N) = (g_A + g_N) - g_N = g_A$.

Dynamics of Capital and Output

Graphical representation

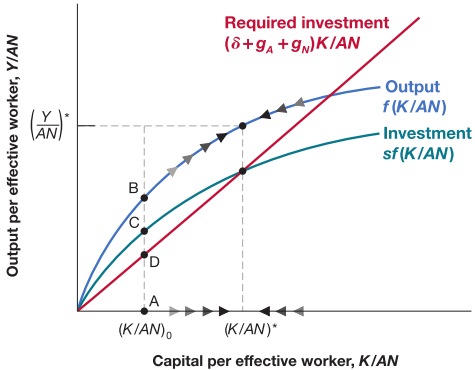

Figura 1: Capital and output dynamics.

Dynamics of Capital and Output

The saving rate and output

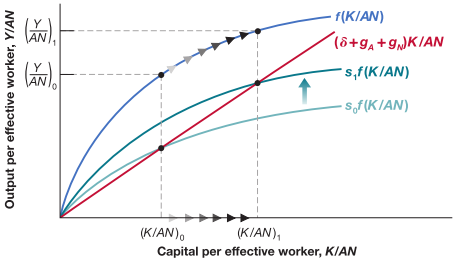

Figura 2: The effects of different saving rates.

- The rate of output growth in the steady state remains unaffected by the saving rate.
- However, the saving rate does influence the steady-state level of output *per* effective worker.
- Increases in the saving rate initially result in a growth rate exceeding the steady-state rate fo a period.

The Solow Residual

- The growth in output due to both labor and capital growth is expressed as $(\alpha g_N + (1-\alpha) g_K)$, where α represents the share of labor in output (or the share of wages in GDP).
- The rate of growth of total factor productivity, commonly known as the Solow residual, quantifies the impact of technological progress as:

$$\text{residual} = g_Y - (\alpha g_N + (1-\alpha) g_K)$$

- Additionally, the Solow residual equals the product of the share of labor and the rate of technological progress:

$$\text{residual} = \alpha g_A$$

References

- Blanchard, O. (2017). *Macroeconomics. Global Edition.* (7^{th} ed.). Routledge.